# FORMULA SHEET

You may use this formula sheet during the Advanced Transport Phenomena course and it should contain all formulas you need during this course. Note that the weeks are numbered from 1.1 to 2.7. Part 1.1 to 1.7 contain all the formulas that were needed in the basic course 'The basics of Transport Phenomena' and part 2.1 to 2.7 contain **new** formulas used in 'Advanced Transport Phenomena'.

## General formulas:

| | |
|---|---|
| Newton's 2nd law of motion. | $F = m * a$ |
| Kinetic energy. | $E_k = \frac{1}{2} m v^2$ |
| Gravitational energy. | $E_g = m * g * h$ |
| Angular velocity of circular motion, where T is the period of the motion. | $\omega = \frac{2\pi}{T}$ |
| Ideal gas law. R is the Gas constant. | $pV = nRT$ |
| Density in mass per unit volume. | $\rho = \frac{m}{V}$ |
| Specific heat: Heat needed to heat an object by 1 degree Celsius. Units are $\frac{J}{kg*K}$. | $C = \frac{Q}{m\Delta T}$ |
| The conversion of going from Celsius to Kelvin. It is important to note that negative temperatures do not exist on the Kelvin scale, while they do for the Celsius scale, so when calculating with absolute temperatures, use Kelvin. In relative calculations where you take a temperature difference, it doesn't matter since Kelvin and Celsius are the same scale, except they are shifted. | $T_K = T_{°C} + 273{,}15$ |
| The radius of a circle, where r is the radius (half the diameter) of the circle. | $s = 2\pi r$ |
| The area of a circle. | $A = \pi r^2$ |
| The volume of a sphere. | $V = \frac{4}{3}\pi r^3$ |

# Constants

| | |
|---|---|
| $N = 6.022 \cdot 10^{23}\ molecules$ | The number of molecules in a mole, called Avogadro's Constant. |
| $R = 8.315 \frac{J}{mol*K}$ | The gas constant |
| $\sigma = 5.67 \cdot 10^{-8}\ \frac{W}{m^2K^4}$ | The Stefan-Boltzmann constant |

# Quantities &Units

| | | |
|---|---|---|
| Mass | $m$ | kg |
| Time | $t$ | s |
| Volume | $V$ | $m^3$ |
| Velocity | $v$ | m/s |
| Density | $\rho$ | $Kg/m^3$ |
| Diameter | D or d | m |
| Force | $F$ | N |
| Temperature | $T$ | K |
| Pressure | $p\ or\ P$ | Pa |
| Mass flow | $\phi$ | Kg/s |
| Diffusion coefficient | $D$ | $m^2/s$ |
| Internal energy | $U$ | J |
| Heat | $Q$ | J |

| | | |
|---|---|---|
| Work | $W$ | Nm |
| Total energy | $E$ | J |
| Area | $A$ | $m^2$ |
| Heat transfer coefficient | $h$ | $W/(m^2 \cdot K)$ |
| Thermal conductivity | $\lambda$ | $W/(m \cdot K)$ |
| Specific heat | $C_p$ | $J/(kg \cdot K)$ |
| Drag coefficient | $C_D$ | - |
| Thermal diffusivity | $a$ | $m^2/s$ |
| Viscosity | $\eta$ | $Pa \cdot s$ |
| Mass transfer coefficient | $k$ | m/s |
| Specific energy dissipation | e | J/kg |
| Shear stress | τ | Pa |
| Wavelength | $\lambda$ | m |

## Air at 20 °C:

| | |
|---|---|
| Density: | 1.205 $kg/m^3$ |
| Heat capacity: | 1.007 kJ/(kg K) |
| Prandtl: | 0.713 |
| Thermal diffusivity: | $2.119 * 10^{-5}\ m^2/s$ |
| Viscosity: | $1.82 * 10^{-5}$ Pa s |

## Water at 20 °C:

| | |
|---|---|
| Density: | 998.23 $kg/m^3$ |
| Heat capacity: | 4.1850 kJ/(kg K) |
| Prandtl: | 7.01 |
| Thermal diffusivity: | $0.143 * 10^{-6}\ m^2/s$ |
| Viscosity: | $1.002 * 10^{-3}$ Pa s |

## WEEK 1.1:

| | |
|---|---|
| The general balance equation. | $\frac{d}{dt} = in - out + production$ |

## WEEK 1.2:

| | |
|---|---|
| Total energy balance | $\frac{dE}{dt} = \phi_{m,in} * \left\{U + \frac{p}{\rho} + \frac{1}{2}v^2 + gh\right\}_{in} - \phi_{m,out} * \left\{U + \frac{p}{\rho} + \frac{1}{2}v^2 + gh\right\}_{out}$ |
| First law of Thermodynamics, where $\Delta W$ is the net work done on the system. | $\Delta U = \Delta Q + \Delta W$ |
| The thermal energy balance in a steady state without energy change. | $0 = \phi_m(u_{in} - u_{out}) + \phi_q + \phi_m e_{fr}$ |
| The mechanical energy balance. | $0 = \phi_m\left(\frac{(v_{in}^2 - v_{out}^2)}{2} + g(h_{in} - h_{out}) + \frac{(p_{in} - p_{out})}{\rho} + \phi_w - \phi_m E_{fr}\right)$ |
| Bernoulli's equation: Neglects all friction and heat production. $h$ is height. | $\frac{p}{\rho} + \frac{v^2}{2} + gh = constant$ |
| Bernoulli's Principle: The energy per unit volume before is the same as the energy per unit volume after. | $P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$ |

## WEEK 1.3:

| | |
|---|---|
| Reynolds number, where $\rho_f$ is the density of the fluid, $v_r$ is the relative velocity, D is the diameter and $\mu$ is the viscosity of the fluid | $Re = \frac{\rho_f v_r D}{\mu}$ |
| The drag force. $C_D$ is the drag coefficient, A is the frontal area, v is the relative velocity. | $F_D = C_D A * \frac{1}{2}\rho_f v_r^2$ |
| Stokes' law: The drag force on a sphere with a low Reynolds number ($Re < 1$). | $F_D = 3\pi D\mu v_r$ |

## WEEK 1.4:

| | |
|---|---|
| Fourier's law, the transfer of heat. $\lambda$ is the material conductivity, $\Delta x$ is the thickness, A is the area, $\Delta T$ is the difference in temperature. | $\phi_q = \lambda A * \frac{\Delta T}{\Delta x}$ |
| Fick's law of diffusion, analogous to Fourier's law. $D$ is the diffusion coefficient, A is the area and $\frac{dC_a}{dx}$ is the change in concentration over x. | $\phi_m = -D * A * \left(\frac{dc_A}{dx}\right)$ |

## WEEK 1.5:

| | |
|---|---|
| Newton's law of cooling. $h$ is the heat transfer coefficient. | $\phi_q = h \cdot A \cdot \Delta T$ |
| Nusselt number. Used to make h dimensionless. | $Nu = \frac{D \cdot h}{\lambda}$ |
| Mass transfer coefficient, where $Sh$ is the Sherwood number, analogous to Nusselt number. $\Delta x$ is the size of the object, also called $D$ sometimes. | $k = Sh \cdot \frac{\mathrm{D}}{\Delta x}$ |

## WEEK 1.6:

| | |
|---|---|
| Thermal diffusivity. $\lambda$ is thermal conductivity, $\rho$ is material density, $C_p$ is specific heat. | $a = \frac{\lambda}{\rho \cdot C_p}$ |
| Penetration depth. Only valid while penetration theory still holds, for $\sqrt{\pi a t} < \frac{D}{2}$, where D is the size of the sheet being penetrated by heat. | $x_p = \sqrt{\pi a t}$ |
| Fourier number. | $Fo = \frac{at}{D^2}$ |
| Nusselt number for penetration theory. | $Nu = \sqrt{\frac{1}{\pi Fo}}$ |

## WEEK 1.7:

No new formulas this week! ☺

## Week 2.1:

| | |
|---|---|
| The general microbalance equation. Where β is the dependent variable of interest | $\frac{d\beta}{dt} = \phi_\beta\|_x - \phi_\beta\|_{x+dx} + P_\beta$ |

## Week 2.2:

| | |
|---|---|
| Momentum balance | $\frac{d}{dt}(m \cdot v_x) = \phi_{m,in} \cdot v_{x,in} - \phi_{m,out} \cdot v_{x,out} + \sum F_x$ |
| Fanning pressure drop equation | $\Delta p = 4f \cdot \frac{L}{D} \cdot \frac{1}{2} \cdot \rho \cdot < v >^2$ |
| Hydraulic diameter, S is the wetted perimeter. | $D_h = \frac{4A}{S}$ |
| The fanning friction factor for the laminar regime: Re < 2000 | $4f = \frac{64}{Re}$ |
| The fanning friction factor for the turbulent regime (formula of Blasius): 4000 < Re < $10^5$ | $4f = 0.316\, Re^{-1/4}$ |
| The specific energy dissipation is modelled as the sum of dissipation in pipelines parts and appendage parts | $e_{diss} = \sum_i (e_{fr})_i + \sum_j (e_L)_j$ |
| Specific energy dissipation in appendages for turbulent flow | $e_L = K_L \cdot \frac{1}{2} \cdot < v >^2$ |

**Gate valve**

| | open | 3/4 | 1/2 | 1/4 |
|---|---|---|---|---|
| $K_L$ | 0.2 | 0.9 | 4.5 | 24 |

**Kink**

| α | 40 | 60 | 80 | 90 | 100 | 120 | 140 | 160 |
|---|---|---|---|---|---|---|---|---|
| $K_L$ | 2.43 | 1.86 | 1.26 | 0.98 | 0.74 | 0.36 | 0.14 | 0.05 |

# WEEK 2.3:

## DIMENSIONLESS NUMBERS:

| | | |
|---|---|---|
| Greatz number: Fraction of conductive over convective heat transfer | $Gz = \frac{aL}{d^2 v}$ | |
| Grashof number: Fraction of buoyancy forces over viscous forces | $Gr = \frac{d^3 g}{v^2} \gamma \Delta T$ | |
| Lewis number: Fraction of thickness of thermal boundary layer over mass transfer boundary layer | $Le = \frac{a}{D}$ | (D is the mass diffusion constant) |
| Peclet (heat) number: Fraction convective heat transfer over conductive heat transfer | $Pe = \frac{vd}{a}$ | |
| Peclet (mas) number: Fraction convective mass transfer over diffusive mass transfer | $Pe = \frac{vd}{D}$ | (D is the mass diffusion constant) |
| Prandtl number: Fraction of hydrodynamic boundary layer thickness over thermal boundary layer thickness | $\mathrm{Pr} = \frac{\nu}{a}$ | (ν is the kinematic viscosity) |
| Schmidt number: Fraction of hydrodynamic boundary layer over mass transfer boundary layer | $Sc = \frac{\nu}{D}$ | (ν is the kinematic viscosity, D is the mass diffusion constant) |

## DIMENSIONLESS CORRELATIONS FOR HEAT TRANSFER:

| | |
|---|---|
| Laminar flow in tubes: $Gz < 0.05$ | $Nu = 1.08\, Gz^{-1/3}$ and $< Nu > = 1.62\, Gz^{-1/3}$ |
| Laminar flow in tubes: $Gz > 0.1$ | $Nu = < Nu > = 3.66$ |
| Turbulent flow in tubes: $Re > 10^4$ and $Pr \geq 0.7$ | $< Nu > = 0.027\, Re^{0.8} \cdot Pr^{0.33}$ |
| Flat plate parallel to flow: $Re < 3 \cdot 10^5$ | $Nu = 0.332\, Re^{1/2} \cdot Pr^{1/3}$ |
| Long cylinders perpendicular to the flow: $10 < Re < 10^4$ and $Pr > 0.7$ and $Pe >> 1$ | $< Nu > = 0.57\, Re^{1/2} \cdot Pr^{1/3}$ |
| Long cylinders perpendicular to the flow: $Re > 10^4$ and $Pr > 0.7$ | $< Nu > = 0.027\, Re^{0.8} \cdot Pr^{0.33}$ |
| Flow around spheres: $10 < Re < 10^4$ and $Pr > 0.7$ and $Pe >> 1$ | $< Nu > = 2 + 0.66\, Re^{1/2} \cdot Pr^{1/3}$ |

**DIMENSIONLESS CORRELATIONS FOR MASS TRANSFER:**

| | |
|---|---|
| Laminar flow in tubes:<br>Gz < 0.05 | $Sh = 1.08\, Gz^{-1/3}$ and $< Sh > = 1.62\, Gz^{-1/3}$ |
| Laminar flow in tubes:<br>Gz > 0.1 | $Sh = < Sh > = 3.66$ |
| Turbulent flow in tubes:<br>$Re > 10^4$ and $Sc \geq 0.7$ | $< Sh > = 0.027\, Re^{0.8} \cdot Sc^{0.33}$ |
| Flat plate parallel to flow:<br>$Re < 3 \cdot 10^5$ | $Sh = 0.332\, Re^{1/2} \cdot Sc$ |
| Long cylinders perpendicular to the flow:<br>$1 < Re < 10^4$ and $Sc > 0.7$ and $Pe >> 1$ | $< Sh > = 0.42\, Sc^{1/5} + 0.57\, Re^{1/2} \cdot Sc^{1/3}$ |
| Flow around spheres:<br>$10 < Re < 10^4$ and $Sc > 0.7$ and $Pe >> 1$ | $< Sh > = 2 + 0.66\, Re^{1/2} \cdot Sc^{1/3}$ |

**OTHER FORMULAS:**

| | |
|---|---|
| Sieder and Tate correction, this equation is used in situations with a viscosity gradient in turbulent pipe flow | $Nu = 0.027\, Re^{0.8} \cdot Pr^{0.33} \cdot (\frac{\mu}{\mu_s})^{0.14}$ |
| The Chilton and Colburn relations combines heat and mass flow coefficients | $k = \frac{h}{\rho\, C_p} Le^{-2/3}$ |

# WEEK 2.4:

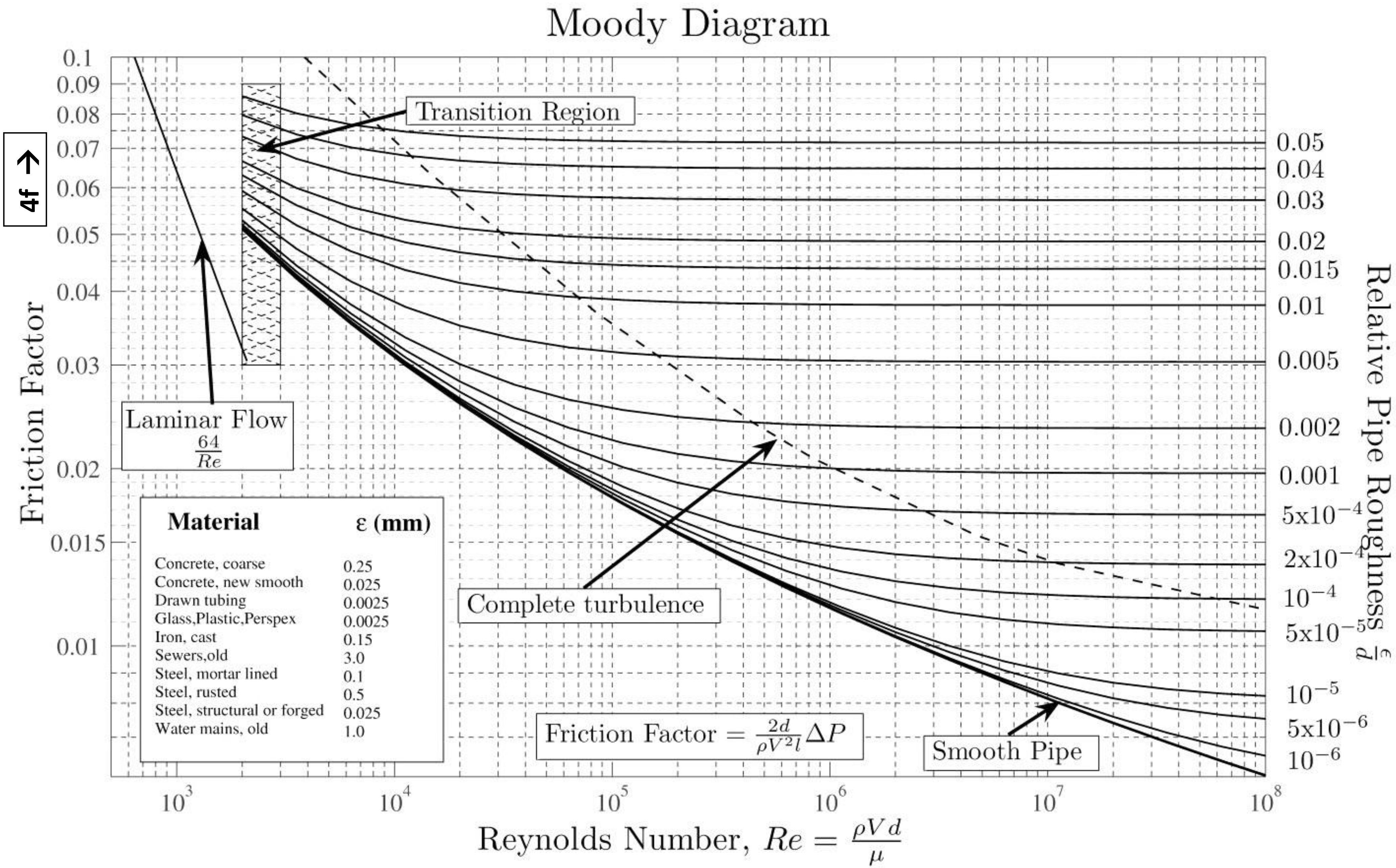
| | |
|---|---|
| The partition coefficient, in which you may assign phases to superscript 1 and 2 | $m = \frac{c^1}{c^2}$ |
| Henry's Law: with p the partial pressure, H the henry's coefficient and y the fraction dissolved in the liquid | $p_A = H_A \cdot y_A$ |

## WEEK 2.5:

| | |
|---|---|
| Shear stress in Newtonian fluids | $\tau_{yx} = -\mu \frac{dv_x}{dy}$ |
| Shear stress for liquids that follow the power law (Ostwald – De Waele model) | $\tau_{yx} = -K \left\vert \frac{v_x}{dy} \right\vert^{n-1} \cdot \frac{dv_x}{dy}$ |
| Shear stress for Bingham liquids | $\vert\tau_{yx}\vert - \tau_0 = \mu \left\vert \frac{dv_x}{dy} \right\vert$ for $\vert\tau_{yx}\vert \geq \tau_0$<br>$\frac{dv_x}{dy} = 0$ for $\vert\tau_{yx}\vert < \tau_0$ |
| Shear stress for visco-elastic fluids, where $\lambda$ is a elasticity parameter | $\tau_{yx} + \lambda \frac{d\tau_{yx}}{dt} = -\mu \frac{dv_x}{dy}$ |
| Hagen-Poiseuille law is used to calculate flow rates from velocity profiles in tubes | $\phi_v = \int_0^R v_x(r)\, 2\pi\, r\, dr$ |

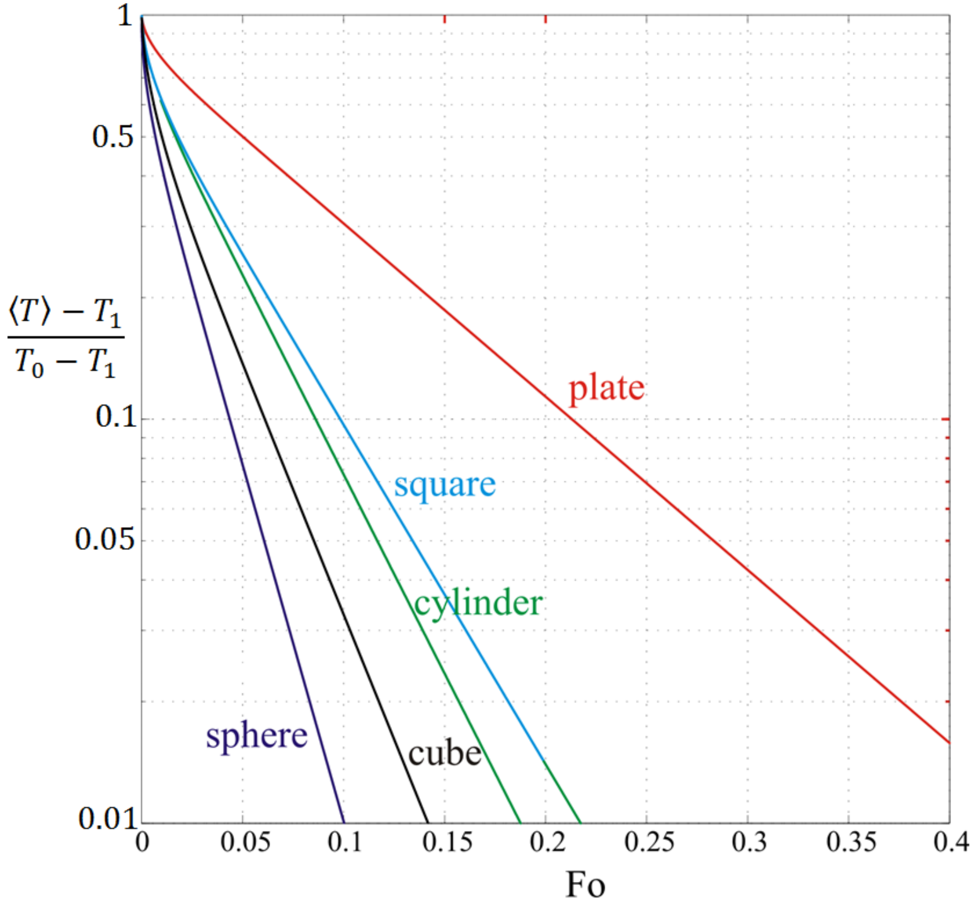
## WEEK 2.6:

| | |
|---|---|
| Stefan-Boltzman Law for grey radiators. Note if e = 1 the object is a black radiator | $\phi_q" = e\, \sigma\, T^4$ |
| Wiens Law that relates the temperature of a radiator to its maximum in radiation wavelength | $\lambda_{max}\, T = 2.898 \cdot\ 10^{-3}\ m \cdot K$ |
| Heat radiation with the help of visibility factors | $\phi_{net,1\rightarrow 2} = F_{1\rightarrow 2} A_1 \sigma T_1^4 - F_{2\rightarrow 1} A_2 \sigma T_2^4$ |

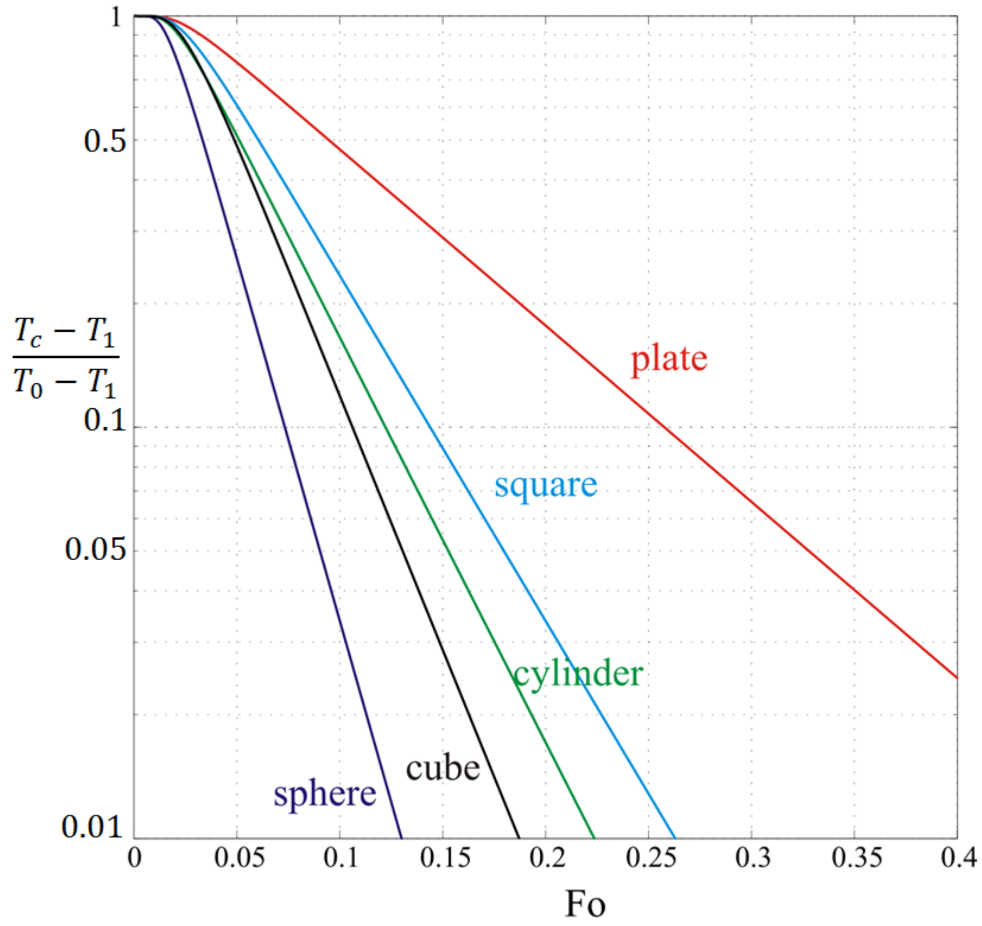


Moody Diagram
4f →
Friction Factor
0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.015
0.01
Transition Region
Laminar Flow
$\frac{64}{Re}$
Material
ε (mm)
Concrete, coarse 0.25
Concrete, new smooth 0.025
Drawn tubing 0.0025
Glass,Plastic,Perspex 0.0025
Iron, cast 0.15
Sewers,old 3.0
Steel, mortar lined 0.1
Steel, rusted 0.5
Steel, structural or forged 0.025
Water mains, old 1.0
Complete turbulence
Friction Factor $= \frac{2d}{\rho V^2 l}\Delta P$
Smooth Pipe
0.05
0.04
0.03
0.02
0.015
0.01
0.005
0.002
0.001
$5x10^{-4}$
$2x10^{-4}$
$10^{-4}$
$5x10^{-5}$
$10^{-5}$
$5x10^{-6}$
$10^{-6}$
Relative Pipe Roughness $\frac{\epsilon}{d}$
$10^3$
$10^4$
$10^5$
$10^6$
$10^7$
$10^8$
Reynolds Number, $Re = \frac{\rho V d}{\mu}$

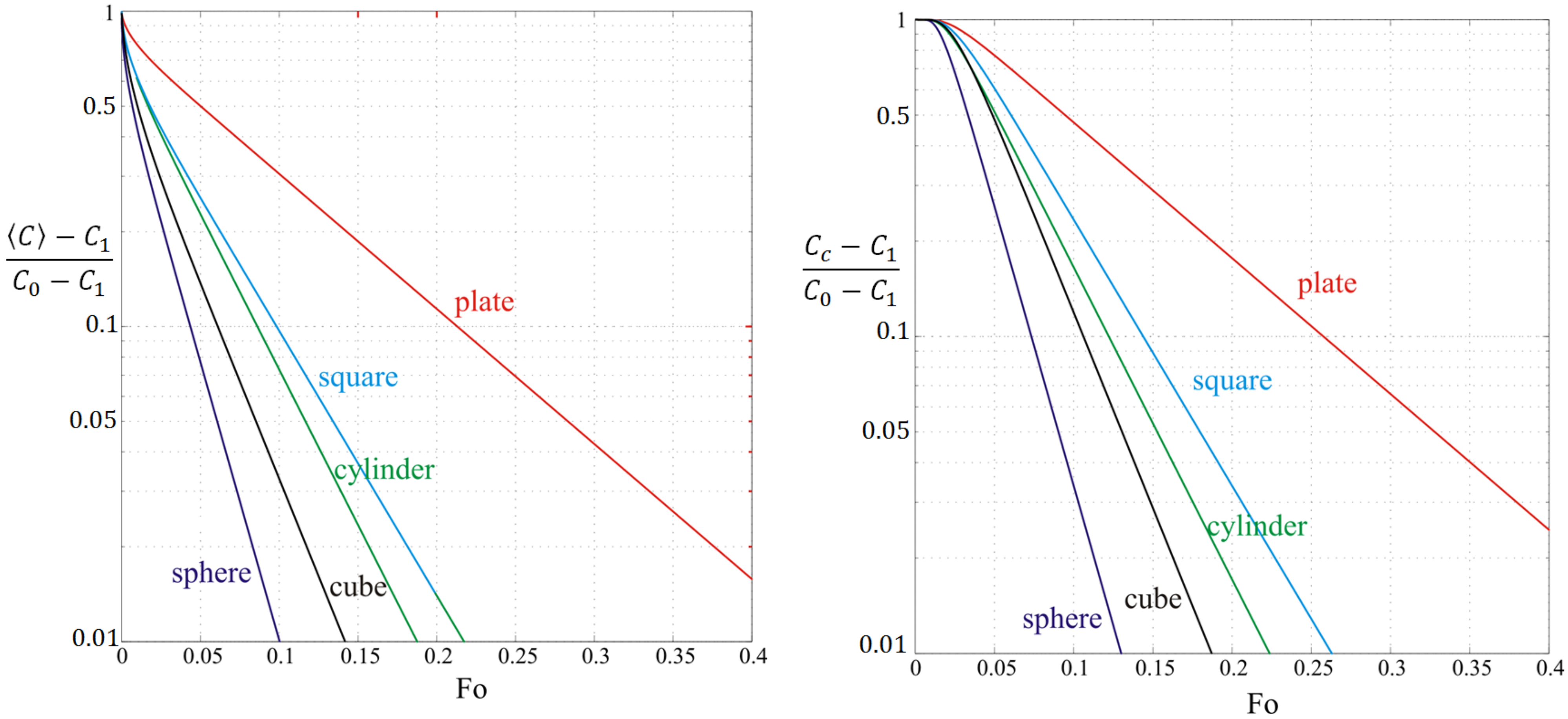
**Graphs**:



1
0.5
0.1
0.05
0.01
$\frac{\langle T \rangle - T_1}{T_0 - T_1}$
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
Fo
plate
square
cylinder
sphere
cube

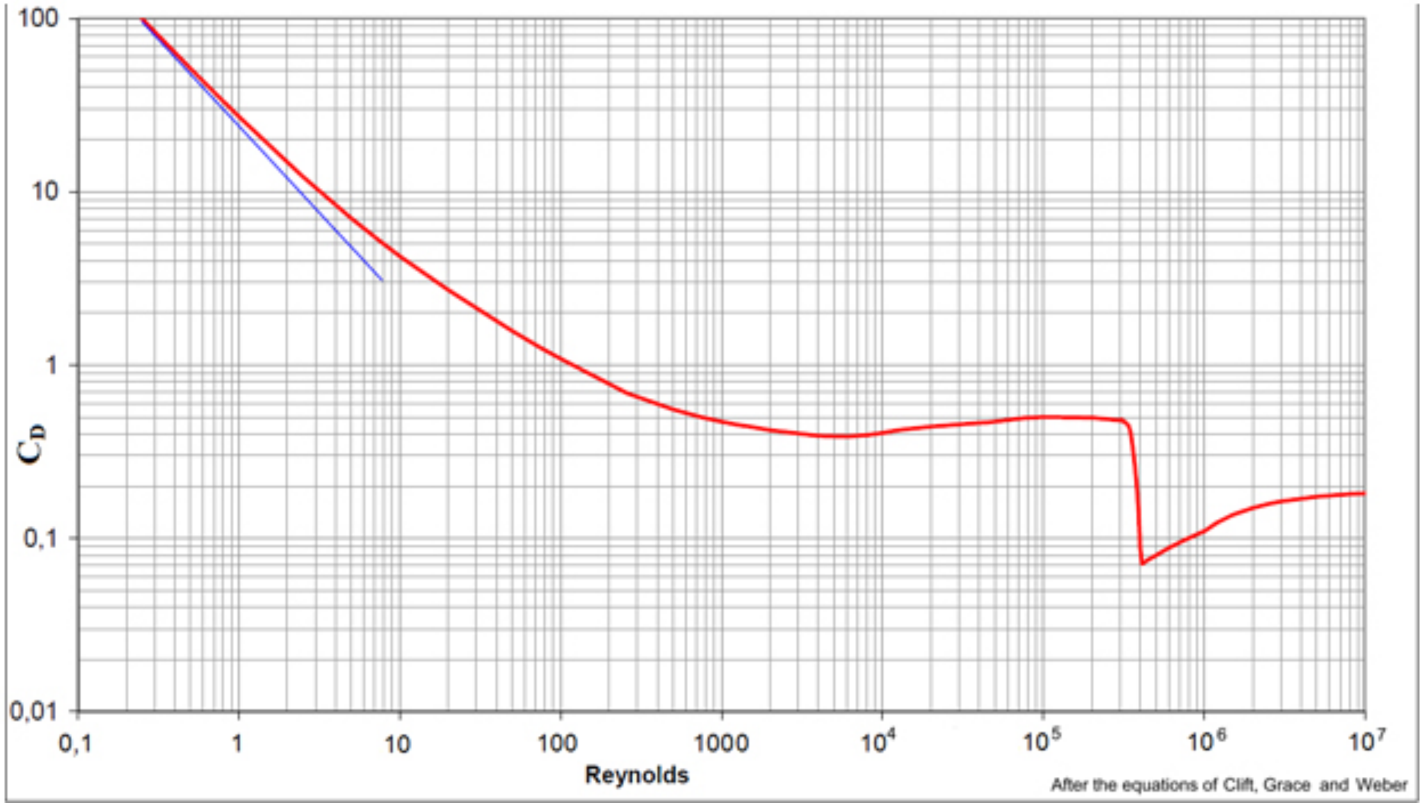



1
0.5
0.1
0.05
0.01
$\frac{T_c - T_1}{T_0 - T_1}$
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
Fo
plate
square
cylinder
cube
sphere



$\frac{\langle C \rangle - C_1}{C_0 - C_1}$
1
0.5
0.1
0.05
0.01
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
Fo
plate
square
cylinder
cube
sphere
$\frac{C_c - C_1}{C_0 - C_1}$
1
0.5
0.1
0.05
0.01
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
Fo
plate
square
cylinder
cube
sphere

**FOR A SPHERE**



100
10
1
0,1
0,01
$C_D$
0,1
1
10
100
1000
$10^4$
$10^5$
$10^6$
$10^7$
Reynolds
After the equations of Clift, Grace and Weber