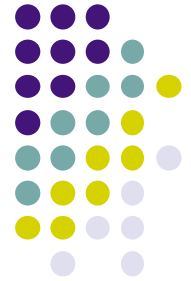


# Definition of a list



- A list is a **recursive** data type: we define it in terms of itself
  - Recursion is used both for computations and data!
  - We need to know this when we write functions on lists
- A list is either an empty list or a pair of an element followed by another list
  - This definition is recursive because it defines lists in terms of lists. There is no infinite regress because the definition is used constructively to build larger lists from smaller lists.
- Let's introduce a formal notation

# Syntax definition of a list

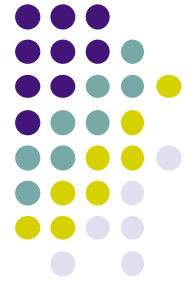


- Using an **EBNF grammar rule** we write:

$$\langle \text{List } T \rangle ::= \text{nil} \mid T \text{ '}' \langle \text{List } T \rangle$$

- This defines the textual representation of a list
- EBNF = Extended Backus-Naur Form
  - Invented by John Backus and Peter Naur
  - $\langle \text{List } T \rangle$  represents a list of elements of type  $T$
  - $T$  represents one element of type  $T$
- Be careful to distinguish between  $\mid$  and  $\text{'}$  : the first is part of the grammar notation (it means “or”), and the second is part of the syntax being defined

# Some examples of lists



- According to the definition (if T is integers):

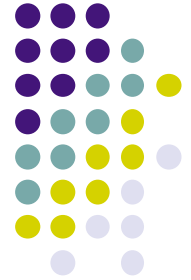
nil

10 | nil

10 | 11 | nil

10 | 11 | 12 | nil

- What about the bracket notation we saw before?
  - It is not part of the recursive definition of lists; it is an extra called *syntactic sugar*



# Type notation

- `<Int>` represents an integer; more precisely, it is **the set of all syntactic representations of integers**
- `<List <Int>>` represents the set of all syntactic representations of lists of integers
- `T` represents the set of all syntactic representations of values of type `T`; we say that `T` is **a type variable**
  - Do not confuse a **type variable** with an **identifier** or a **variable in memory!** Type variables exist only in grammar rules.

# Don't confuse a thing and its representation



René Magritte, *La trahison des images*, 1928-29, oil, Los Angeles County Museum of Art, Los Angeles.

1234

- This is not a pipe. It is a digital display of a photograph of a painting of a pipe (thanks to Belgian surrealist René Magritte for pointing this out!).
- This is not an integer. It is a digital display of a visual representation of an integer using numeric symbols in base 10.