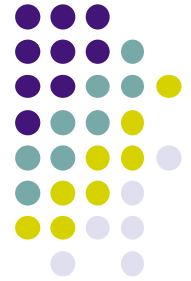


The functional kernel language



- Now we have seen all the concepts in the functional paradigm that we will use
 - We can define its **full kernel language**
- We will use this kernel language to understand exactly what a functional program does
 - We have used it to see **why list functions are tail-recursive**
 - We will use it as **part of the formal semantics** (in lesson 6)
- Each time we introduce a new paradigm in the course we will define its kernel language
 - Each extends the functional kernel language with a new concept

The functional kernel language (in part)



- $\langle s \rangle ::=$ **skip**
 - | $\langle s \rangle_1 \langle s \rangle_2$
 - | **local** $\langle x \rangle$ **in** $\langle s \rangle$ **end**
 - | $\langle x \rangle_1 = \langle x \rangle_2$
 - | $\langle x \rangle = \langle v \rangle$
 - | **if** $\langle x \rangle$ **then** $\langle s \rangle_1$ **else** $\langle s \rangle_2$ **end**
 - | **proc** $\{ \langle x \rangle \langle x \rangle_1 \dots \langle x \rangle_n \}$ $\langle s \rangle$ **end**
 - | $\{ \langle x \rangle \langle y \rangle_1 \dots \langle y \rangle_n \}$
 - | **case** $\langle x \rangle$ **of** $\langle p \rangle$ **then** $\langle s \rangle_1$ **else** $\langle s \rangle_2$ **end**
- $\langle v \rangle ::=$ $\langle \text{number} \rangle$ | $\langle \text{list} \rangle$ | ...
- $\langle \text{number} \rangle ::=$ $\langle \text{int} \rangle$ | $\langle \text{float} \rangle$
- $\langle \text{list} \rangle, \langle p \rangle ::=$ nil | $\langle x \rangle$ | $\langle x \rangle$ ' | $\langle \text{list} \rangle$

This is what we have seen so far

The functional kernel language (in part)



- $\langle s \rangle ::=$ **skip**
 - | $\langle s \rangle_1 \langle s \rangle_2$
 - | **local** $\langle x \rangle$ **in** $\langle s \rangle$ **end**
 - | $\langle x \rangle_1 = \langle x \rangle_2$
 - | $\langle x \rangle = \langle v \rangle$
 - | **if** $\langle x \rangle$ **then** $\langle s \rangle_1$ **else** $\langle s \rangle_2$ **end**
 - | **proc** $\{ \langle x \rangle \langle x \rangle_1 \dots \langle x \rangle_n \}$ $\langle s \rangle$ **end**
 - | $\{ \langle x \rangle \langle y \rangle_1 \dots \langle y \rangle_n \}$
 - | **case** $\langle x \rangle$ **of** $\langle p \rangle$ **then** $\langle s \rangle_1$ **else** $\langle s \rangle_2$ **end**

This is what we have seen so far;
it needs *two changes* to become
the full kernel language of the
functional paradigm

1. Procedure
declarations
(should be values)

- $\langle v \rangle ::=$ $\langle \text{number} \rangle$ | $\langle \text{list} \rangle$ | ...
- $\langle \text{number} \rangle ::=$ $\langle \text{int} \rangle$ | $\langle \text{float} \rangle$
- $\langle \text{list} \rangle, \langle p \rangle ::=$ nil | $\langle x \rangle$ | $\langle x \rangle$ ' | $\langle \text{list} \rangle$

2. Compound types (should be more than lists only)

The functional kernel language (in part)



- $\langle s \rangle ::=$ skip
| $\langle s \rangle_1 \langle s \rangle_2$
| **local** $\langle x \rangle$ **in** $\langle s \rangle$ **end**
| $\langle x \rangle_1 = \langle x \rangle_2$
| $\langle x \rangle = \langle v \rangle$
| **if** $\langle x \rangle$ **then** $\langle s \rangle_1$ **else** $\langle s \rangle_2$ **end**
| ~~**proc** $\{ \langle x \rangle \langle x \rangle_1 \dots \langle x \rangle_n \} \langle s \rangle$ **end**~~
| $\{ \langle x \rangle \langle y \rangle_1 \dots \langle y \rangle_n \}$
| **case** $\langle x \rangle$ **of** $\langle p \rangle$ **then** $\langle s \rangle_1$ **else** $\langle s \rangle_2$ **end**
- $\langle v \rangle ::=$ $\langle \text{number} \rangle$ | $\langle \text{procedure} \rangle$ | $\langle \text{list} \rangle$ | ...
- $\langle \text{number} \rangle ::=$ $\langle \text{int} \rangle$ | $\langle \text{float} \rangle$
- $\langle \text{procedure} \rangle ::=$ **proc** $\{ \$ \langle x \rangle_1 \dots \langle x \rangle_n \} \langle s \rangle$ **end**
- $\langle \text{list} \rangle, \langle p \rangle ::=$ nil | $\langle x \rangle$ | $\langle x \rangle$ ' | $\langle \text{list} \rangle$

1. Procedures are values in memory (like numbers and lists)

The functional kernel language **(complete)**



- $\langle s \rangle ::=$ **skip**
 - | $\langle s \rangle_1 \langle s \rangle_2$
 - | **local** $\langle x \rangle$ **in** $\langle s \rangle$ **end**
 - | $\langle x \rangle_1 = \langle x \rangle_2$
 - | $\langle x \rangle = \langle v \rangle$
 - | **if** $\langle x \rangle$ **then** $\langle s \rangle_1$ **else** $\langle s \rangle_2$ **end**
 - | $\{ \langle x \rangle \langle y \rangle_1 \dots \langle y \rangle_n \}$
 - | **case** $\langle x \rangle$ **of** $\langle p \rangle$ **then** $\langle s \rangle_1$ **else** $\langle s \rangle_2$ **end**
- $\langle v \rangle ::=$ $\langle \text{number} \rangle$ | $\langle \text{procedure} \rangle$ | ~~$\langle \text{list} \rangle$~~ | $\langle \text{record} \rangle$
- $\langle \text{number} \rangle ::=$ $\langle \text{int} \rangle$ | $\langle \text{float} \rangle$
- $\langle \text{procedure} \rangle ::=$ **proc** $\{ \$ \langle x \rangle_1 \dots \langle x \rangle_n \}$ $\langle s \rangle$ **end**
- $\langle \text{record} \rangle, \langle p \rangle ::=$ $\langle \text{lit} \rangle$ | $\langle \text{lit} \rangle (\langle f \rangle_1 : \langle x \rangle_1 \dots \langle f \rangle_n : \langle x \rangle_n)$

2. Records subsume lists

The functional kernel language (complete)



- $\langle s \rangle ::=$ **skip**
 - | $\langle s \rangle_1 \langle s \rangle_2$
 - | **local** $\langle x \rangle$ **in** $\langle s \rangle$ **end**
 - | $\langle x \rangle_1 = \langle x \rangle_2$
 - | $\langle x \rangle = \langle v \rangle$
 - | **if** $\langle x \rangle$ **then** $\langle s \rangle_1$ **else** $\langle s \rangle_2$ **end**
 - | $\{ \langle x \rangle \langle y \rangle_1 \dots \langle y \rangle_n \}$
 - | **case** $\langle x \rangle$ **of** $\langle p \rangle$ **then** $\langle s \rangle_1$ **else** $\langle s \rangle_2$ **end**
- $\langle v \rangle ::=$ $\langle \text{number} \rangle$ | $\langle \text{procedure} \rangle$ | $\langle \text{record} \rangle$
- $\langle \text{number} \rangle ::=$ $\langle \text{int} \rangle$ | $\langle \text{float} \rangle$
- $\langle \text{procedure} \rangle ::=$ **proc** $\{ \$ \langle x \rangle_1 \dots \langle x \rangle_n \}$ $\langle s \rangle$ **end**
- $\langle \text{record} \rangle, \langle p \rangle ::=$ $\langle \text{lit} \rangle$ | $\langle \text{lit} \rangle (\langle f \rangle_1 : \langle x \rangle_1 \dots \langle f \rangle_n : \langle x \rangle_n)$

Procedure values and records are important basic types. They allow, for example, to define all the concepts of object-oriented programming.

Kernel language of the functional paradigm



- $\langle s \rangle ::=$ **skip**
 - | $\langle s \rangle_1 \langle s \rangle_2$
 - | **local** $\langle x \rangle$ **in** $\langle s \rangle$ **end**
 - | $\langle x \rangle_1 = \langle x \rangle_2$
 - | $\langle x \rangle = \langle v \rangle$
 - | **if** $\langle x \rangle$ **then** $\langle s \rangle_1$ **else** $\langle s \rangle_2$ **end**
 - | $\{ \langle x \rangle \langle y \rangle_1 \dots \langle y \rangle_n \}$
 - | **case** $\langle x \rangle$ **of** $\langle p \rangle$ **then** $\langle s \rangle_1$ **else** $\langle s \rangle_2$ **end**
- $\langle v \rangle ::=$ $\langle \text{number} \rangle$ | $\langle \text{procedure} \rangle$ | $\langle \text{record} \rangle$
- $\langle \text{number} \rangle ::=$ $\langle \text{int} \rangle$ | $\langle \text{float} \rangle$
- $\langle \text{procedure} \rangle ::=$ **proc** $\{ \$ \langle x \rangle_1 \dots \langle x \rangle_n \}$ $\langle s \rangle$ **end**
- $\langle \text{record} \rangle, \langle p \rangle ::=$ $\langle \text{lit} \rangle$ | $\langle \text{lit} \rangle (\langle f \rangle_1 : \langle x \rangle_1 \dots \langle f \rangle_n : \langle x \rangle_n)$

Three ways to understand languages

