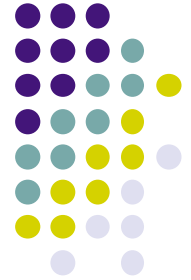


Trees

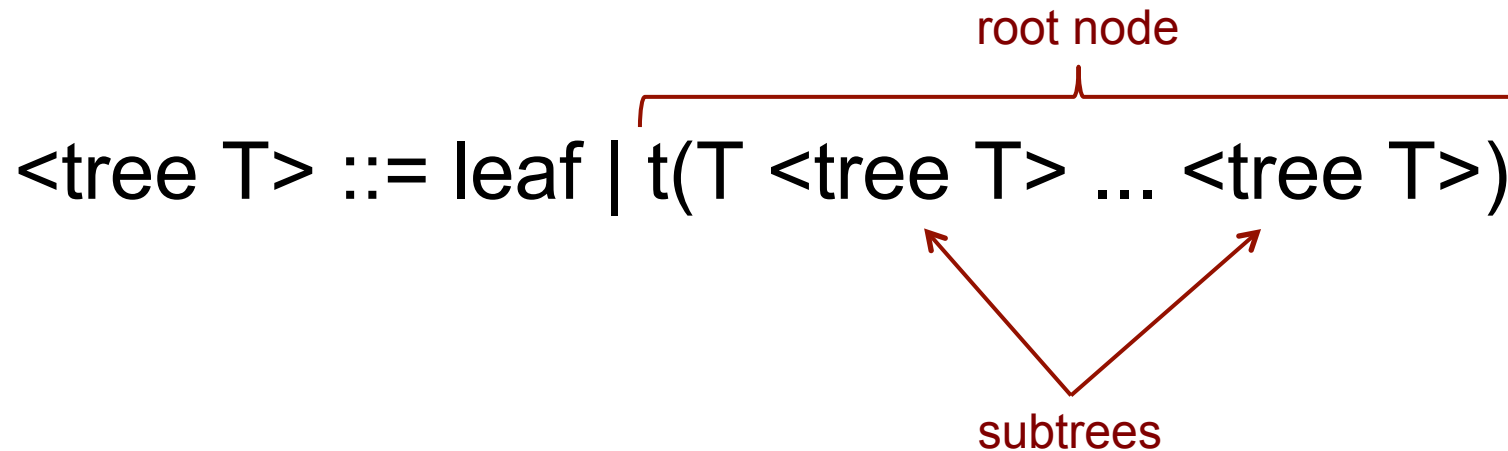


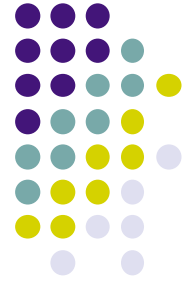
- Trees are the **second most important data structure** in computing, next to lists
 - Trees are extremely useful for efficiently organizing information and performing many kinds of calculations
- Trees illustrate well **goal-oriented programming**
 - Many tree data structures are based on a global property, that must be maintained during the calculation
- In this lesson we will define trees and use them to store and look up information
 - We will define **ordered binary trees** and algorithms to add information, look up information, and remove information

Trees



- A tree is a **recursive structure**: it is either an empty tree (called a leaf) or an element and a set of trees

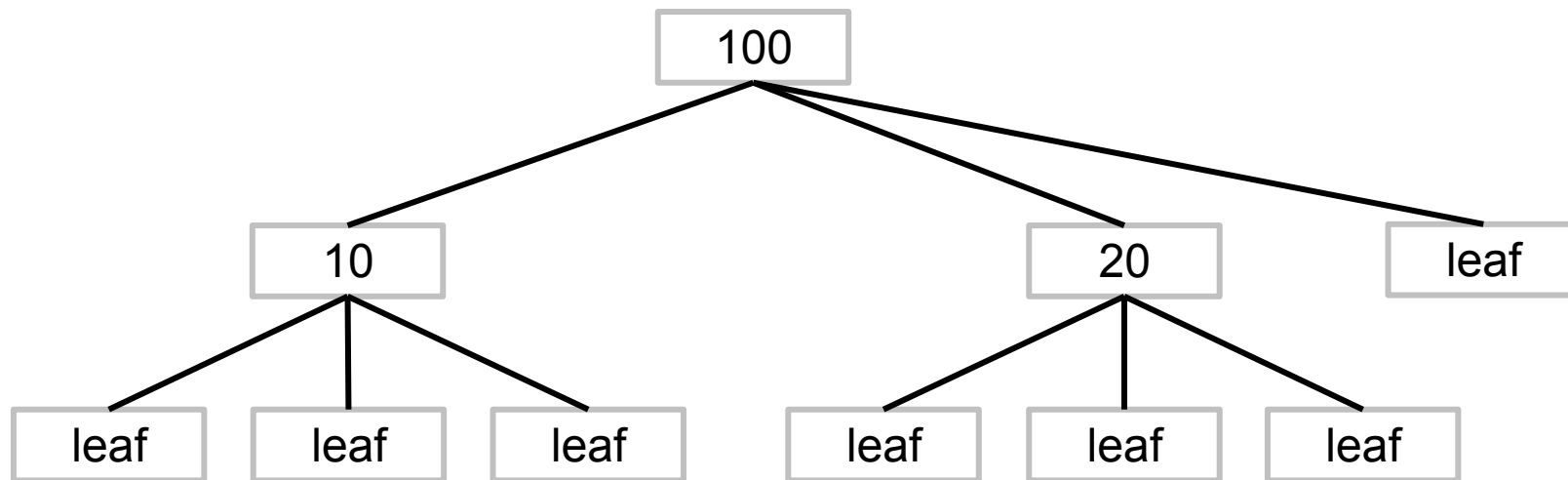




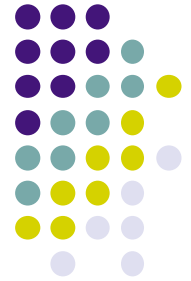
Example tree

- **declare**

$T = t(100 \ t(10 \ \text{leaf} \ \text{leaf} \ \text{leaf}) \ t(20 \ \text{leaf} \ \text{leaf} \ \text{leaf}) \ \text{leaf})$



Trees compared to lists



- A tree is a recursive structure: it is either an empty tree (called a leaf) or an element and a set of trees

$\langle \text{tree } T \rangle ::= \text{leaf} \mid t(T \langle \text{tree } T \rangle \dots \langle \text{tree } T \rangle)$

$\langle \text{list } T \rangle ::= \text{nil} \mid ' | '(T \langle \text{list } T \rangle)$

Notice the
similarity with lists!