

# Best case, average case, and worst case



? What does it mean to say  $f(n)$  is a function of input size  $n$  if there can be *many* different inputs with same size  $n$  (e.g., if the input is a list)?

!  $f(n)$  might be different for different input lists!

- So we need to take some kind of *average*

- There are three standard ways to do this: best case, average case, and worst case
  - **Best case**: take only inputs of size  $n$  with smallest time
  - **Worst case**: take only inputs of size  $n$  with largest time
  - **Average case**: take inputs of size  $n$  according to some probability distribution (which must be given)

# Example of best, average, and worst



- Let's use the function {FirstNegative L} that takes a list of integers and returns the position of the first negative:
  - For example, {FirstNegative [5 ~8 6 7]} returns 2
- **Best case:** We only give lists whose first element is negative. Then  $f_{best}(n) \in O(1)$
- **Worst case:** We only give lists with all elements positive except the last. Then  $f_{worst}(n) \in O(n)$
- **Average case:**
  - If each element has independent probability 0.5 to be negative, then  $f_{average}(n) \in O(1)$
  - If the *position* of the first negative element in the list is uniformly distributed from 1 to n, then  $f_{average}(n) \in O(n)$